



Grades Eight Through Twelve

Introduction

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion. To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Standards are provided for algebra I, geometry, algebra II, trigonometry, mathematical analysis, linear algebra, probability and statistics, Advanced Placement probability and statistics, and calculus. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

Table 1, “Mathematics Disciplines, by Grade Level,” reflects typical grade-level groupings of these disciplines in both integrated and traditional curricula. The lightly shaded region reflects the minimum requirement for mastery by all students. The dark shaded region depicts content that is typically considered elective but that should also be mastered by students who complete the other disciplines in the lower grade levels and continue the study of mathematics.

Table 1
Mathematics Disciplines, by Grade Level

<i>Discipline</i>	<i>Grades</i>				
	<i>Eight</i>	<i>Nine</i>	<i>Ten</i>	<i>Eleven</i>	<i>Twelve</i>
Algebra I					
Geometry					
Algebra II					
Probability and Statistics					
Trigonometry					
Linear Algebra					
Mathematical Analysis					
Advanced Placement Probability and Statistics					
Calculus					

Many other combinations of these advanced subjects into courses are possible. What is described in this section are standards for the academic content by discipline; this document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

When students delve deeply into mathematics, they gain not only conceptual understanding of mathematical principles but also knowledge of, and experience with, pure reasoning. One of the most important goals of mathematics is to teach students logical reasoning. The logical reasoning inherent in the study of mathematics allows for applications to a broad range of situations in which answers to practical problems can be found with accuracy.

By grade eight, students' mathematical sensitivity should be sharpened. Students need to start perceiving logical subtleties and appreciate the need for sound mathematical arguments before making conclusions. As students progress in the

study of mathematics, they learn to distinguish between inductive and deductive reasoning; understand the meaning of logical implication; test general assertions; realize that one counterexample is enough to show that a general assertion is false; understand conceptually that although a general assertion is true in a few cases, it is not true in all cases; distinguish between something being proven and a mere plausibility argument; and identify logical errors in chains of reasoning.

Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline students master at more advanced levels.

Algebra I

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

- 1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:
 - 1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

- 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

- 3.0 Students solve equations and inequalities involving absolute values.

- 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.

- 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

- 6.0 Students graph a linear equation and compute the x - and y -intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

- 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

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- 8.0** Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
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- 9.0** Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
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- 10.0** Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.
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- 11.0** Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
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- 12.0** Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
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- 13.0** Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.
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- 14.0** Students solve a quadratic equation by factoring or completing the square.
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- 15.0** Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
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- 16.0** Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

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- 17.0** Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
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- 18.0** Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
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- 19.0** Students know the quadratic formula and are familiar with its proof by completing the square.
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- 20.0** Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
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- 21.0** Students graph quadratic functions and know that their roots are the x -intercepts.
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- 22.0** Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x -axis in zero, one, or two points.
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- 23.0** Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
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- 24.0** Students use and know simple aspects of a logical argument:
- 24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
 - 24.2 Students identify the hypothesis and conclusion in logical deduction.
 - 24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

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- 25.0** Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:
- 25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
 - 25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
 - 25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

Geometry

The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their ability to construct formal, logical arguments and proofs in geometric settings and problems.

- 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

- 2.0 Students write geometric proofs, including proofs by contradiction.

- 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

- 4.0 Students prove basic theorems involving congruence and similarity.

- 5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

- 6.0 Students know and are able to use the triangle inequality theorem.

- 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

- 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

- 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.

- 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

- 11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.

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- 12.0** Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.
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- 13.0** Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.
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- 14.0** Students prove the Pythagorean theorem.
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- 15.0** Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.
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- 16.0** Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.
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- 17.0** Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.
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- 18.0** Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.
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- 19.0** Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.
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- 20.0** Students know and are able to use angle and side relationships in problems with special right triangles, such as 30° , 60° , and 90° triangles and 45° , 45° , and 90° triangles.
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- 21.0** Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.
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- 22.0** Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections.